MATH 504 HOMEWORK 3

Due Wednesday, February 17.

Problem 1. *Suppose that κ is strongly inaccessible. Show that:*

1. *α is an ordinal iff $V_\kappa \models$ "α is an ordinal".*
2. *α is a cardinal iff $V_\kappa \models$ "α is a cardinal".*

Problem 2. *Suppose that κ is strongly inaccessible. Show that:*

1. *α is a regular cardinal iff $V_\kappa \models$ "α is a regular cardinal".*
2. *α is strongly inaccessible iff $V_\kappa \models$ "α is strongly inaccessible".*

Note: the above problem shows that if κ is the least inaccessible cardinal, then $V_\kappa \models$"there are no inaccessible cardinals".

Problem 3. *Suppose that κ is inaccessible. Show that $|V_\kappa| = \kappa$ and V_κ satisfies the Replacement axiom, i.e. show that if $f : X \to V_\kappa$ is a function from a set $X \in V_\kappa$, then $f \in V_\kappa$.*

Problem 4.
1. *Show that if $\aleph_\omega < 2^\omega$, then $(\aleph_\omega)^\omega = 2^\omega$.*
2. *If κ is singular strong limit, show that $2^\kappa = \kappa^{\mathrm{cf}(\kappa)}$. (Hint: compute $2^{<\kappa}$.)*

Problem 5. *Show that $\aleph_\omega^{\omega_1} = 2^{\omega_1} \cdot \aleph_\omega^\omega$*